Dynamics of yarn tension on knitting machines

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Abstract

The submitted paper reports a study of investigation of the dynamics of yarn tension, i.e. to the time dependence of the yarn tension on large-diameter circular knitting machines (LCKM). The special feature of this study is the fact that predictions are made for the course of the yarn tension during a single stitch-forming process.

Only the yarn tension in the area of the yarn feeder and the yarn guide eye can be measured. Some results of such experimental investigations have already been published [1,2]. In this paper, additional results of theoretical model calculations are presented. In particular, by these calculations it will be possible to determine the structure of the yarn tensile force at the knitting point, i.e. in the region between knitting needles.

The calculation of the yarn tension required a numerical solution of a system of non-linear differential equations. The special difficulty lies in the formulation of the non-equilibrium state of the running yarn, especially in the formulation of the robbing-back effect.

Introduction

The yarn tension of a yarn running into a large-diameter circular knitting machine (LCKM) is an important technological parameter. In particular the dynamic properties of the yarn tensile force contain information about the quality of the knitting process and, in addition, about knitting machine failures [1,2]. Finding out the time dependence of the yarn tension in the region between the needles is of interest, as well. The order of the yarn tension values that occur during the stitch-forming process determines the damage of the yarn as well as of the needles and is therefore also of great importance to the quality of the knitting process.

First theoretical views about the knitting process with consideration of the robbing-back-effect appeared in the sixties. However, at that time the status of the computer technique did not allow for numerical calculations in today's sense. One of the first fundamental papers in this field comes from Knapton and Munden [3]. In [3] already the important relationships between knitting parameters and knitwear characteristics are described. The calculation of the yarn tension is based on the equilibrium states which are derived graphically. A relatively simple view of the situation at a knitting point is given by Merrit [4]. Here, the yarn which is delivered to the knitting point is distributed in the region between the needles. The delivery takes place in such a way that exactly as much yarn is supplied as is consumed in the region of the knitting point. A calculation of the yarn speed at the needle hooks, also when robbing-back occurs, is possible. A possibility for predicting the yarn tension is suggested. Ghosh and Banerjee [5,6] above all experimentally investigate the stitch size as a function of the needle motion and the take-down forces of the tubular knitted fabric. The extent of the robbing-back-effect is derived from the stitch size. The possibility of a theoretical prediction of the robbing-back-effect is answered in the negative [5] or formulated as a current task [6], respectively. In the paper by Bauer [7] the calculation of both the stitch size and the yarn tension between the area of the yarn feeder and the knitting point is shown. The calculation is carried out analytically without consideration of robbing-back.

A measurement of the yarn tension in the region of the knitting needles on an LCKM appears impossible at this time. In order to get information about the yarn tension and its time dependence, model calculations are useful. The special difficulty of such calculations, however, is that there are non-stationary states of the yarn parameters. Non-stationary states of running yarn usually are not taken into account because the stationary state is assumed. The submitted work is a suggestion for the calculation such nonstationary states. The calculation is made using a special needle track geometry on an LCKM. Some examples of the results of these calculations are described.
The aim of the paper presented here consists of describing the situation at a knitting point in more detail. The findings can be used in order to find possibilities for the improving the knitting process. Further, the results should be used to find new equipment and supervising principles for the detection of failures during the knitting process.

**Definitions**

**Knitting machine / Knitting point**

The starting point of the considerations is a situation at a knitting point shown in Figure 1. The horizontal distances between the needles are equal $l_t$. $v_M$ is the peripheral speed of the LCKM. The distance between knitting needles changes at a speed $v$ which is a superposition of $v_M$ and the speed of the needles in the needle channels $v_{N<}$.

![Diagram of the yarn course on a LCKM for model calculations](http://www.autex.org/v1n2/2272_00.pdf)

**Figure 1**: Diagram of the yarn course on a LCKM for model calculations

The following symbols are used:

- distance of feedwheel unit and yarn guiding eye: $L_1$
- distance of yarn guiding eye and first knitting needle: $L_2$
- distance between needles: $L_3 \ldots L_6$
- yarn tension between the area of feedwheel unit and yarn guiding eye: $F_E$
- yarn tension between the area of yarn guiding eye and first knitting needle: $F_1$
- yarn tension between the area of the knitting needles: $F_2 \ldots F_6$
- yarn tension in the yarn storage of the feeder: $F_{E}$
- speed of the needles: $v_{N1} \ldots v_{N4}$
- peripheral speed of the LCKM: $v_M$
- feeding rate of the feedwheel unit: $v_E$

**Cam track**

The distance between the yarn guiding eye and the first knitting needle as well as the distance between needles depends on the shape of the needle track. The geometry of the needle track can be assumed at choice.
Yarn

The yarn is assumed to show linear elastic behaviour according to

\[ F = \varepsilon \cdot E \]  

(1)

where \( F \) is the yarn tensile force, the elongation of the yarn, and \( E \) the (linear) elastic constant of the yarn. The mass of stretched yarn can be calculated according to

\[ m = l \cdot (\rho A) \]  

(2)

where \( l \) is the length of the elongated yarn, \( \rho \) is the mass density of yarn, and \( A \) is the cross section of the yarn. The mass per length of unstretched yarn \( m_0 / l_0 = (\rho_0 A_0) \) is usually designated as yarn count in textile production. Based on these suppositions the mass per length is

\[ \rho A = (\rho_0 A_0) \cdot \frac{E}{F + E} \]  

(3)

Feedwheel unit

The delivery rate of the feedwheel unit is \( v_E \). The relation between peripheral speed \( v_M \) of the LCKM and the synchronous feeding rate \( v_E \) of the feedwheel unit is assumed to be

\[ v_E = 4 \cdot v_M \]  

(4)

This is a realistic relation in a knitting process. It means that a single stitch uses a yarn length \( 4 \cdot l \). Using any other ratio \( v_E/v_M \) is possible. The yarn tension in the yarn storage of the feeder is, as mentioned above, \( F_E \).

Yarn Guiding Eye

The yarn guiding eye has a cylindrical form. One can assume a multiplicative relationship between friction forces of the yarn running over the needle hooks according to equation (5)

\[ F_2 = F_1 \cdot e^{\alpha \mu} \]  

(5)

where \( \alpha \) is the wrapping angle and \( \mu \) the coefficient of the friction of the yarn-eye.

Robbing-back-Effect

If the yarn runs over a needle hook one can assume that the yarn tension in front of and behind the needle can be described in accordance with equation (5). If there is robbing-back, i.e. when a direction reversal of the yarn course at the needle hook occurs, equation (5) does not describe the situation.

\[ ??? - \text{undefined (there does not exist any mathematical description of the time dependence of yarn tensile force when a direction-reversal takes place).} \]
In Figure 2 the described situation is shown schematically. The yarn runs over the needle hook in the one direction, then a direction-reversal takes place. The values determined the equation (5) are drawn in the diagram in Figure 2. The situation during the time between these states is not clear. The substantial aim of the following considerations is to formulate this problem mathematically.

![Diagram showing yarn over needle hook with forces and masses](image)

**Figure 3: Schematic diagram of mass invariance**

Figure 3 shows the yarn course over a needle hook schematically. The yarn length on the left side of the needle is $L_1$, the yarn length on the right side is $L_2$. It comes to a balance of the forces under dynamic conditions because the yarn flows from the one side into the other side. In which way this occurs is very difficult to say because a complicated connection between the Newton law and the effect of the bonding friction and the gliding friction occurs here. For the integration of this situation into the theoretical model the formulation of the mass invariance is important because it is always valid. Using equation (3), one can formulate the mass invariance

$$m_1 + m_2 = L_1 \cdot \frac{E}{F_1 + E} + L_2 \cdot \frac{E}{F_2 + E} = \text{const.}$$  \hspace{1cm} (6)

The equilibrium state can be calculated in this way. The yarn mass on the left side of the needle before mass flow is $m_1$

$$m_1 = L_1 \cdot \frac{E}{F_1 + E}$$  \hspace{1cm} (7)

After the balance of the forces, the same yarn tensile force occurs at both sides of the needle. Now, a mass $m_1 + \ddot{m}$ is in the area $L_1$.

$$m_1 + \ddot{m} = L_1 \cdot \frac{E}{F + E}$$  \hspace{1cm} (8)

The yarn mass which flows over the needle hook is $\ddot{m}$.

$$\ddot{m} = L_1 \cdot \frac{E}{F + E} \cdot \frac{E}{F_1 + E}$$  \hspace{1cm} (9)

Using an always valid relation $F$, $F_1$, $F_2 \gg E$ yields to

$$\ddot{m} = \frac{L_1}{E} \left( F_1 - F \right)$$  \hspace{1cm} (10)

and

$$\ddot{m} = \frac{L_2}{E} \left( F - F_2 \right)$$  \hspace{1cm} (11)
With equations (10) and (11) the equilibrium force $F$ and the transported mass $\Delta m$ can be written

$$F = \frac{L_1 F_1 + L_2 F_2}{L_1 + L_2} \quad (12)$$

$$\Delta m = \frac{L_1 L_2}{L_1 + L_2} \cdot \frac{F_1 - F_2}{E} \quad (13)$$

The equations (12) and (13) indicate the physical limits for the transport of yarn mass and yarn tension. Depending upon the conditions for bonding friction and gliding friction, $F_1$ can be reduced and $F_2$ can be increased up to $F$. If the transient yarn mass flow ends, i.e. if the condition $F_1 = F_2 \cdot \exp (\alpha \mu)$ is fulfilled a similar calculation yields to the result

$$\Delta m = \frac{L_1 L_2}{L_1 \cdot \exp(\alpha \mu) + L_2} \cdot \frac{F_1 - F_2 \cdot \exp(\alpha \mu)}{E} \quad (14)$$

The problem of the description of a yarn course with direction reversal is not solved thereby. The limits of $\Delta m$ and $F$ can be given. However, the time dependence of these parameters cannot. All efforts to derive the time dependence of $\Delta m$ and $F$ from the basic principles failed due to the expenditure and the uncertainty of the description of the needle-yarn-friction.

For model calculations, it appears useful to consider 3 cases described as follows:

**Case 1:**

$F_1$ and $F_2$ fulfil the demand of equation (5), i.e. $F_1 = F_2 \cdot \exp(\alpha \mu)$

This is the case of a dynamic equilibrium, the yarn speed has the correct value. The yarn mass per time, which is transported over the needle hook, has the correct value. It must not be modified

$$\frac{dm}{dt} \bigg|_{t+\Delta t} = \frac{dm}{dt} \bigg|_{t} \quad (15)$$

**Case 2:**

$F_1$ and $F_2$ do not fulfil the demands of equation (5), the value of the ratio $F_1/F_2$ is too large, i.e. $F_1 > F_2 \cdot \exp(\alpha \mu)$

The yarn mass flow is too large, it must be reduced. The amount of the reduction is proportional to the mass difference according to equation (14) and a factor $1/T$.

$$\frac{dm}{dt} \bigg|_{t+\Delta t} = \frac{dm}{dt} \bigg|_{t} \cdot \frac{1}{T} \cdot \frac{L_1 L_2}{L_1 \cdot \exp(\alpha \mu) + L_2} \cdot \frac{F_1 - F_2 \cdot \exp(\alpha \mu)}{E} \quad (16)$$

It is not clear which value must have the constant factor $1/T$. A decision can be realised only experimentally. One must determine experimentally with which time constant the transient process takes place. Such measuring possibilities are not available. Therefore, for the model calculations a value $T = 0.01$ ms is assumed. This means, it is assumed that changes of the yarn speed which occur due to different yarn tensions, are realised in a time interval of 0.01 ms. This time constant $T$ is so defined that the time to establish the mass equilibrium is clearly faster than the time for the formation of a single stitch.

**Case 3:**

$F_1$ and $F_2$ do not fulfil the demands of equation (5), the value of the ratio $F_1/F_2$ is too small, i.e. $F_1 < F_2 \cdot \exp(\alpha \mu)$

The yarn speed must decrease and become zero because a stationary yarn run is not possible in this situation. It is therefore assumed that the reduction of the yarn speed varies exponentially with time i.e. $dm/dt \sim \exp(-t/\tau)$. The time constant $\tau$ has its lowest value (0.01 s), if $F_1=F_2$. This means that the reduction of the yarn speed to the zero value occurs fastest when the forces at both sides of the needle hook are equal. In the case of $F_1>F_2$, the time constant $\tau$ is increased. It becomes infinitely large, if the condition $F_1 = F_10 = F_2 \cdot \exp(\alpha \mu)$ is fulfilled. The yarn speed is not reduced in this case, it remains constant.

$$\frac{dm}{dt} \bigg|_{t+\Delta t} = \frac{dm}{dt} \bigg|_{t} \cdot \exp \left\{ \frac{F_10 - F_1}{F_10 - F_2} \cdot \frac{\Delta t}{\tau} \right\} \quad (17)$$
Modelling of the total stitch-forming process

Model calculation of the described yarn-feeding and stitch-forming process is based on the definitions and designations shown in Figure 1. The constructional parameters of the knitting machine and the needle track are independent parameters. From this, the wrapping angles of the yarn at the yarn guiding eye and the needle hooks can be calculated as a function of time. Further, the elastic behaviour of the yarn and the peripheral speed of the LCKM must be given.

The mathematical formulation of the total process is, as already stated, based on the invariance of the yarn mass. To each yarn length \( L \), can be formulated:

\[
\frac{dm}{dt} \bigg|_{\text{fed to } L} = \frac{E}{F_E + E} \cdot v_E + \frac{dm}{dt} \bigg|_{\text{out to }}
\]

With this supposition can be written:

Mass of yarn which is delivered by the yarn feeding unit

\[
\frac{dm}{dt} \bigg|_{\text{feed}} = \frac{E}{F_E + E} \cdot v_E \quad (18)
\]

Mass of yarn in the area of feeder and yarn guide eye, \( L_1 \):

\[
\frac{dm}{dt} \bigg|_{\text{eye } \rightarrow L_2} = L_1 \cdot \frac{E}{(F_1 + E)^2} \cdot \frac{dF_1}{dt} + \frac{dm}{dt} \bigg|_{\text{eye } \rightarrow L_2} \quad (19)
\]

Mass of yarn in the area of yarn guide eye and first knitting needle, \( L_2 \):

\[
\frac{dm}{dt} \bigg|_{\text{eye } \rightarrow L_3} = L_2 \cdot \frac{E}{(F_2 + E)^2} \cdot \frac{dF_2}{dt} + \frac{E}{F_2 + E} \cdot \frac{dL_2}{dt} + \frac{dm}{dt} \bigg|_{L_2 \rightarrow L_3} \quad (20)
\]

Mass of yarn in the area of knitting needles \( L_3 \), \( L_5 = L_3 \ldots L_5 \):

\[
\frac{dm}{dt} \bigg|_{L(v-1) \rightarrow L_v} = L_v \cdot \frac{E}{(F_v + E)^2} \cdot \frac{dF_v}{dt} + \frac{E}{F_v + E} \cdot \frac{dL_v}{dt} + \frac{dm}{dt} \bigg|_{L_v \rightarrow L(v+1)} \quad (21)
\]

Mass in the area of last knitting needle and fabric tube \( L_6 \):

\[
\frac{dm}{dt} \bigg|_{L_5 \rightarrow L_6} = L_6 \cdot \frac{E}{(F_6 + E)^2} \cdot \frac{dF_6}{dt} + \frac{E}{F_6 + E} \cdot \frac{dL_6}{dt} \quad (22)
\]

Equations (18) to (22) describe the yarn tension at a knitting point of a LCKM. The situation is characterised by a system of 6 coupled differential equations. The coupling of the differential equations comes from the conditions for the transport of the yarn mass which is described by the equations (15) - (17). Thus the total process is completely formulated.

Solution of the differential equation system

An analytical solution for the system of the differential equations is not possible. Only numerical procedures are applicable. The numerical solution was carried out by means of the mathematics software "matlab". A program consisting of some sub-programs ("m-files"), was created. For the numerical solution of the system of the differential equations "matlab" uses the Runge-Kutta method.

Results

The calculation of the time dependence of the yarn tension is difficult. The main reason for this is the correct formulation and calculation of the yarn mass transport over the needle hooks and the yarn guide eye. Small
deviations of the yarn tensile forces from the equilibrium state described in equation (5) lead to a strong change in the mass flow rate across the needle hooks or the yarn guiding eye. These strong changes influence the entire yarn run situation, in particular the concrete values of the yarn tension of the individual yarn lengths $L_i$, very sensitively. In order to obtain correct and useful results, the time intervals $\Delta t$ for integration must be chosen to be very small - in the present case $\Delta t$ lies in the order of some $\mu$s.

The following parameters were used to calculate the time dependence of the yarn tension. These are parameters of a real LCKM in our laboratory, so we can compare some calculated yarn tensile forces and experimental results:

- peripheral speed $v_M = 1$ m/s
- feeding rate of the feedwheel unit $v_E = 4$ m/s
- yarn tension in the yarn storage of the feeder $F_E = 4$ cN
- distance of feeder and yarn guiding eye $L_1 = 600$ mm
- distance of yarn guiding eyelet and first knitting needle $L_2 = 5$ mm
- distance between needles $L_\circ$ (is a special function)
- elasticity constant of yarn (according to $F=E$) $E = 500$ cN
- coefficient of friction $\mu = 0.2$

General characteristics of calculated yarn tension curves

Figure 4 shows first an example of the time dependence of $F_1(t)$. This is the yarn tensile force in the area of feeder and yarn guiding eye, measurable in the knitting mill without problems. The yarn tensile force $F_1$ is almost constant. The regular needle operation leads to a slight modulation of $F_1(t)$, which can also be detected in experiment [1,2]. The second diagram shows the yarn tensile force in the area of the knitting needles. It shows that the yarn tensile force rises clearly. However, when robbing-back occurs, the force also may be reduced to small values. The experimental proof of the correctness of this result is desirable, but practically impossible.

![Yarn tension forces under stationary conditions](http://www.autex.org/v1n2/2272_00.pdf)

Figure 4: Yarn tensile forces under stationary conditions

Effect of yarn tension in front of the feeder

During the time interval $\Delta t$ of an individual stitch-forming process yarn material of the length $L_6$ and with the force $F_6$ is taken off the knitting point. The mass $m_{off}$ that is taken off in $\Delta t$ is
\[ m_{\text{off}} = \frac{E}{F_E + E} \cdot L_6 \]  

(24)

During a single stitch-forming process the feeder supplies the yarn mass \( m_{\text{feed}} \).

\[ \begin{aligned} 
    m_{\text{feed}} &= \frac{E}{F_E + E} \cdot v_E \cdot \Delta t 
    
    \end{aligned} \]  

(25)

Mass invariance means \( m_{\text{off}} = m_{\text{feed}} \). From this follows

\[ F_6 = L_6 \cdot \frac{F_E + E}{v_E \cdot \Delta t} \cdot E \]  

(26)

Figure 5 shows the yarn tensile forces \( F_1 \ldots F_6 \) at the end of a stitch-forming process. The proportionality between \( F_E \) and \( F_6 \) can be clearly seen. Fluctuations of the yarn tension \( F_E \) in front of the feeder lead to relatively small fluctuations in the yarn tension \( F_1 \) within the area feeder – yarn guiding eye. An on-line measurement and a monitoring of \( F_1 \) can, therefore, provide only insufficient information about the quality of the yarn supply.

**Figure 5:** Dependence of yarn tensile forces in the area of knitting needles \( F_i \) on yarn tension in front of the feeder \( F_E \)

Effect of friction coefficient

Figure 6 shows the yarn tensile forces \( F_1 \ldots F_6 \) at the end of a stitch-forming process as a function of the coefficient of friction \( \mu \).
It is clearly perceptible that $F_6$ does not depend on $\mu$. This is a consequence of the mass invariance because the removed yarn mass cannot change when $\mu$ changes - in accordance with equation (26). Because of the relation $F_1 = F_2 \cdot \exp\{\alpha \mu\}$ all other yarn tension values become smaller with the rise of the coefficient of friction $\mu$. A too small value of the yarn tension in the area of the feeder and LCKM is not necessarily an excessive yarn supply by the feeder.

A situation dangerous to the stitch-forming process can occur when the yarn friction value increases stepwise. Such a situation occurs, for example when the degree of waxing of the yarn changes. For the model calculations it is assumed that the coefficient of friction of yarn material immediately changes according to

$$
\begin{align*}
\mu &= 0.15 & \text{for } & 0 \leq t < 5 \text{ ms} \\
\mu &= 0.30 & \text{for } & t \geq 5 \text{ ms}.
\end{align*}
$$

The result of the calculation is shown in Figure 7.

The yarn tensions reach stationary values that fulfil the $\exp\{\alpha \mu\}$ law. The rising of $\mu$ leads to an increase of the ratio $(F_{<i}) / (F_j)$, which in its part leads to an increase of the yarn force differences and, thus, to a very strong rise in the yarn tension, in particular at the last knitting needles. Caused by the high value of $F_6$, now a lesser yarn mass than the feeder supplies is led off the knitting point. The consequence of the excess of delivered yarn is a decrease of all yarn tensions. In particular, the value of $F_6$ approximates to the original value of $F_6(t=0)$. This transient process proceeds over some 100 stitch forming-processes and ends with (harmless) stationary values of the yarn tensions according to Figure 6.
**Figure 7:** Time dependence of yarn tensile forces $F_i$ when the coefficient of friction changes: $=0.15 \rightarrow =0.30$

**Conclusions**

The theoretical view on the yarn tensile forces on an LCKM represented here is, of course, a special case. It is quite simply a consequence of the expenditure. The special feature of the calculations presented here consist of the facts that

- **robbing-back** is taken into account,
- calculations of the **structure** of the yarn tension were made.

Under the formulated prerequisites, the presented results are certainly suitable to make qualitatively correct predictions to special questions of the yarn tension during the stitch-forming process.

The numerical calculation of the yarn tensile force processes is extraordinarily complex. The main problem is the numerical stability of the system of differential equations. The smallest variations of the ratio of the yarn tensile forces $(F_{i+1}) / (F_i)$ over a needle hook lead to large changes in the mass flow rate $dm/dt$ over this friction body. The danger of numerical instabilities is obvious. The integration step size $\Delta t$ is controlled by the software itself according to the accuracy. The $\Delta t$ is in the order of microseconds. The calculation of a yarn tensile force curve over 100 stitch-forming processes takes from several minutes to up to one hour on a modern PC.

**References**


